Estimating the time-varying parameters of interests rates models by maximum principle

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An important area of financial mathematics studies the expected returns and volatilities of the price dynamics of stocks, bonds, and interest rates. The stochastic dynamics of stocks, bonds, and interest rates should be correctly specified since misspecification of a model leads to erroneous valuation and hedging. We have to admit that economic conditions change from time to time, so we assume that return and volatility depend on time as well as on price level for some stock or bond. In this case it is reasonable to use a stochastic differential equation with the time-varying parameters as the model for the description of the price dynamics.

It is not easy to describe the time-varying parameters by means of certain functional forms. Flexible models do not assume any specific form of these functions. This data analytic approach called nonparametric regression can be found in statistical literature. However, the direct application of the ideas does not bring desired results. The improvements of the identification procedures were presented in [1, 2, 4]. The main idea of these works was based on the discretization of the stochastic differential equation and further approximation of the parameter functions by constants at the discretization points. It is clear that the accuracy of the estimates depends on the accuracy of the discretization method. To overcome this problem we propose to consider the time-varying parameters as an control functions and solve the identification task as an optimal control problem using the maximum principle [3, 5].

In the paper we present the principles of the identification method construction, show its proficiency and give some illustrations.

References

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