

# *Bayesian ex post evaluation of recursive multi-step-ahead density prediction*

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## **Abstract**

This research is focused on a formal Bayesian method of recursive multi-step-ahead density prediction and its ex post evaluation. Our approach remains in the framework of the standard (classical or orthodox) Bayesian paradigm based on the Bayes factor and on the use of the likelihood-based update. We propose a new decomposition of predictive Bayes factor into the product of partial Bayes factors, for both a finite number of consecutive  $k$ -period-ahead forecasts (where  $k > 1$ ) and the recursive updates of posterior odds ratios based on updated data sets. The first factor in the decomposition is related to the relative  $k$ -period-ahead forecasts ability of models, and the second factor is a measure of the updating effect.

To illustrate usefulness of the measures proposed, we apply the new decomposed predictive Bayes factors to compare forecasting ability of models when the true data generating process (DGP) is known, i.e. using simulated data sets. Taking into account the effect of updating posterior odds ratios leads to the conclusion that the best model is the one with the true process generating data. However, the highest  $k$ -period-ahead forecasting ability (considered alone) can be achieved by other, less adequate models.

Next, we investigate predictive ability of different Vector Error Correction (VEC) models with conditional heteroscedasticity for sets of the US and Polish macroeconomic variables: unemployment, inflation and interest rates. The results show that forecasting ability of the models depends on the forecast horizon as well as on taking into account the update effect.